Signal-to-Noise Ratio Enhancement Based on Wavelet Filtering in Ultrasonic Testing

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Abstract

In ultrasonic non-destructive testing of materials with a coarse-grained structure the scattering from the grains causes backscattering noise, which masks flaw echoes in the measured signal. Several filtering methods have been proposed for improving the signal-to-noise ratio. In this paper we present a comparative study of methods based on the wavelet transform. Experiments with stationary, discrete and wavelet packet de-noising are evaluated by means of signal-to-noise ratio enhancement. Measured and simulated ultrasonic signals are used to verify the proposed de-noising methods. For comparison, we use signal-to-noise ratio enhancement related to fault echo amplitudes and filtering efficiency specific for ultrasonic signals. The best results in our setup were achieved with the wavelet packet de-noising method.

Key words: Ultrasonic testing, Wavelets, Filtering

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1 Introduction

Ultrasonic non-destructive testing (NDT) based on the pulse–echo method is widely used for defect detection in materials. In practical applications for materials with a non-homogeneous or coarse-grained structure, the signal energy is lost due to scattering, so it is often difficult to detect small flaws. There are many centers in coarse-grained materials that can generate echoes that seem to be randomly distributed in time. These echoes are usually referred to as backscattering noise. Several filtering methods have been proposed for improving the signal-to-noise ratio. In this paper we present a comparative study of methods based on the wavelet transform. Experiments with stationary, discrete and wavelet packet de-noising are evaluated by means of signal-to-noise ratio enhancement. Measured and simulated ultrasonic signals are used to verify the proposed de-noising methods. For comparison, we use signal-to-noise ratio enhancement related to fault echo amplitudes and filtering efficiency specific for ultrasonic signals. The best results in our setup were achieved with the wavelet packet de-noising method.

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to as backscattering noise. The typical ultrasonic signal can be written in the form $x(t) = a(t) + n_1(t) + n_2(t)$, where $a(t)$ is received ultrasonic echo, $n_1(t)$ is backscattering noise and $n_2(t)$ is noise caused primarily by electronic circuitry. Both undesirable backscattering noise and noise from electronic circuitry have to be cancelled without suppressing the fault echoes that characterize flaws.

Widely used methods at the present time are split spectrum processing [1] and wavelet based filtering [3,4,6–10]. The results are presented in various forms, so that a direct comparison is very difficult.

Wavelet-based filtering methods are generally non-linear and the behaviour of these filters strongly depends on the input value. It is therefore not possible to compare various filtering techniques through a single signal-to-noise ratio enhancement (SNRE) value. We propose to use an SNRE related to fault echo amplitudes. As a filtering figure of merit, specific for ultrasonic signals, we introduce filtering efficiency, which evaluates both amplitude and shape distortions.

In this paper we compare the wavelet based methods on both synthetic and real signals.

This paper is organized as follows. The second section describes the principles of wavelet transform de-noising methods and threshold estimation. Section 3 evaluates the proposed methods. All comparisons were performed on simulated ultrasonic signals with typical backscattering noise, introduced by M. Gustafsson and T. Stepinski [2]. For the performance evaluation we used the real ultrasonic signal measured on a coarse-grained material for airplane engines. Section 4 contains our conclusions.

2 Wavelet Based Filtering

The wavelet transform is a multiresolution analysis technique that can be used to obtain a time-frequency representation of an ultrasonic signal. In addition to the discrete wavelet transform (DWT), there are many extensions of the basic wavelet transform principle, of which the stationary wavelet transform (SWT) [8] and wavelet packets (WP) [4] are most widely used for de-noising purposes. In general, the de-noising procedure can be described as follows:

- decomposition of the input noisy signal into $N$ levels of the approximations and detailed coefficients, using the selected wavelet transform,
- thresholding of coefficients,
- reconstruction of the signal using approximations and detailed coefficients by means of the inverse transform.
The purpose of the thresholding procedure is to eliminate or suppress small value coefficients which mainly represent the noise content. Standard thresholding methods retain only coefficients exceeding the estimated threshold value. In hard thresholding, coefficients with absolute values lower than the threshold are set to zero, while soft thresholding in addition shrinks the remaining nonzero coefficients toward zero. Soft thresholding avoids problems with spurious oscillations, while hard thresholding typically results in a smaller mean square error. The main problem of wavelet de-noising is the choice of a proper mother wavelet (basis function), thresholding method and threshold value estimator for optimal performance [7].

Fig. 1. Illustration of the noise suppression procedure based on the discrete wavelet transform (DWT), only two decomposition levels are depicted, HP and LP are high-pass resp. low-pass filters, ↑ 2 and ↓ 2 stand for up-sampling resp. down-sampling.

The discrete stationary wavelet transform (SWT) [5] is an undecimated version of DWT. The main idea is to average several detailed coefficients, which are obtained by decomposition of the input signal without downsampling. This approach can be interpreted as a repeated application of the standard DWT method for different time shifts.

The wavelet packets (WP) method [4] is a generalization of wavelet decomposition that offers a larger range of possibilities for signal analysis due to the full decomposition tree. In wavelet packets analysis, a signal is split into approximations and detailed coefficients, then not only the detailed but also the approximation coefficients are split into a second-level approximation and details, and the process is repeated. All coefficients are thresholded. The other steps are similar to the DWT based de-noising method.
3 Results

3.1 Artificial signals

The received ultrasonic signal contains echoes caused by scattering from grains in materials with a non-homogeneous structure. These echoes are called backscattering noise. A second source of noise in the ultrasonic signal is noise from electronic circuitry. The backscattering noise generation used in this work is based on the simple clutter model presented in [2]. We consider noise to be the superimposition of signals coming from grains in the material. Considering the Rayleigh region \((\lambda \gg D)\), where \(\lambda\) is the wavelength and \(D\) is the diameter of the material grain) the frequency response of the material can be expressed [2] by

\[
H_{\text{mat}}(\omega) = \sum_{k=1}^{K_{\text{tot}}} \beta_k \frac{\omega^2}{x_k} \exp(-\alpha_s 2x_k \omega^4) \exp(-i\omega \frac{2x_k}{c_l}),
\]

where \(\alpha_s\) is material attenuation coefficient, \(c_l\) is velocity of the longitudinal waves, \(x_k\) is the grain positions of \(k = 1...K_{\text{tot}}\) number of grains and \(\beta_k\) is a random vector depending on the grain volume. The signal of the backscattering noise in the frequency domain can be expressed [2] by

\[
H_{\text{bn}}(\omega) = H_t(\omega) H_t(\omega) H_{\text{mat}}(\omega).
\]

The \(H_t(\omega)\) occurs twice since the ultrasonic transducer is used as a transmitter and as a receiver in our case. This model was used for generating backscattering noise, see Fig. 2 - right and the corresponding frequency spectrum is depicted in Fig. 2 - left. The measured ultrasonic signal also contains a second

Fig. 2. Backscattering noise - left, frequency spectrum - right.

source of noise, which is caused by electronic circuitry. This source of noise depends on the ultrasonic transducer and ultrasonic instrument that are used.
This electronic noise can be approximated as white noise with a Gaussian amplitude distribution. To construct a real ultrasonic signal, both electronic and backscattering noise were added. Fig. 3 - left represents the typical ultrasonic noise, with the corresponding frequency spectrum in Fig. 3 - right.

To construct an ultrasonic signal in a pulse–echo testing setup, we can add the back-wall echo and the fault echo to the ultrasonic noise. The frequency spectrum of an impulse that has passed the transducer twice and has propagated through a material $2d_{\text{crack}}$ in thickness can be expressed \[2\] as

$$S(\omega) = e^{\alpha s 2d_{\text{crack}}\omega^4} e^{j2d_{\text{crack}}/c_{l}} H_t(\omega) H_f(\omega)$$  \hspace{1cm} (3)$$

The typical ultrasonic signal measured in a clear place on a material with a coarse-grained structure is shown in Fig. 4.

The figure shows the echoes caused by the reflection of grains considered in a grainy material only.
3.2 Signals from Coarse–grained Materials

A set of simulated ultrasonic signals was created. An ultrasonic signal was proposed based on the real ultrasonic signal measured on a coarse-grained material. This material is commonly used in the construction of airplane engines. The basic parameters and coefficients for ultrasonic noise construction have to be used. The material is \( d_{\text{max}} = 10 \text{ mm} \) in thickness. We used an ultrasonic transducer with an operating frequency of 25 MHz, and the returned signal was sampled at 1024 consecutive time instants at 200 MHz sampling frequency. The crack was placed at a depth \( d_{\text{crack}} = 5 \text{ mm} \). In the simulations, the speed of longitudinal sound wave \( c_l \) was set to the value \( 6250 m \cdot s^{-1} \), and the material attenuation coefficient \( \alpha_s = 1 \cdot 10^{-28} \) based on the experimental findings. Based on a microscopic analysis (see Fig. 5) of the material, 200 scatterers were used for clutter generation.

![Fig. 5. Microscopic image of the grainy material.](image)

A simulation of the ultrasonic signal measured on a coarse-grained material was created. The amplitude of the electronic noise was experimentally chosen as 5% of the maximum amplitude of the backscattering noise. The simulated ultrasonic signal with the crack situated in the center of the depth of the material is shown in Fig. 6.

3.3 Performance of de-noising methods

The simulated ultrasonic signal was used to compare different wavelet transform-based de-noising methods. To the simulated ultrasonic signal we added the different amplitudes of the fault echo and performed the three wavelet transform de-noising methods mentioned here.

The following section evaluates wavelet transform de-noising using different
parameters as mother wavelets, threshold rules and threshold levels. For comparison, we used different mother wavelets: Daubechies family of order 4 (db4) and 6 (db6), Symlet of order 6 (sym6), Haar (haar) and the discrete Meyer wavelet (dmey). The decomposition level was experimentally set to 4. A higher level of decomposition does not improve the de-noising performance. These mother wavelets with a comparison of the ultrasonic echo are shown in Fig. 7.

![Simulated ultrasonic signal with a fault echo.](image)

Fig. 6. Simulated ultrasonic signal with a fault echo.

All these mother wavelets have different properties, the most important of which are presented in Tab. 1.

![Examples of mother wavelets with projected echo.](image)

Fig. 7. Examples of mother wavelets with projected echo - a) Daubechies 4 (db4), b) Daubechies 6 (db6), c) Haar (haar), d) discrete Meyer (dmey)

All these mother wavelets were used for de-noising of ultrasonic signals. In case of the thresholding rule, many rules have been suggested for wavelet de-noising [6]. The most commonly used methods are hard and soft thresholding. In addition, others papers present the compromising method [12] and the custom method [11]. They overcome the disadvantages of the hard- and soft-thresholding method. The compromising method is defined as follows:
### Table 1

Important properties of mother wavelets

<table>
<thead>
<tr>
<th>Mother wavelet</th>
<th>Discrete transform</th>
<th>Symmetry</th>
<th>Approximation FIR filters</th>
<th>Exact reconstruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daubechies</td>
<td>•</td>
<td>–</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Symlet</td>
<td>•</td>
<td>–</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>Haar</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>discrete Meyer</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
</tbody>
</table>

\[
\hat{T}_{ij}^{\text{comp}} = \begin{cases} 
0 : |T_{ij}| < T \\
\text{sign}(T_{ij})(T_{ij} - \alpha T) : |T_{ij}| \geq T.
\end{cases}
\]  

(4)

Thus it is a compromise between hard and soft thresholding, where the difference is caused by constant \( \alpha \). If \( \alpha = 0 \), hard thresholding can be considered, and if \( \alpha = 1 \), the equation corresponds to soft thresholding. The custom method is defined as

\[
\hat{T}_{ij}^{\text{custom}} = \begin{cases} 
T_{ij} - \text{sign}(T_{ij})(1 - \alpha)T : |T_{ij}| > T \\
0 : |T_{ij}| \leq \tau \\
\alpha T \left(\frac{|T_{ij}|-\tau}{T-\tau}\right)^2 \left\{ (\alpha - 3) \left(\frac{|T_{ij}|-\tau}{T-\tau}\right) + 4 - \alpha \right\} : \text{otherwise.}
\end{cases}
\]  

(5)

The principle of custom and compromising thresholding rules is illustrated in Fig. 8.

![Fig. 8. Principle of custom and compromising thresholding - a) custom thresholding \( T = \text{const.}, 0 < \alpha < 1 \), b) compromising thresholding \( \alpha = \text{const.}, 0 < \tau < T \)](image_url)

Soft thresholding (see Fig. 8) is not suitable for an ultrasonic signal, because in addition to the noise the fault echo amplitude is also suppressed due to the reduction of the remaining nonzero coefficients toward zero. The amplitude of
fault echo is usually used for defect sizing, consequently the amplitude lowering is undesirable. In our simulations only hard, compromising and custom thresholding will be considered.

When the threshold rule is selected, the threshold level should be finally derived. Standard methods do not produce efficient results for typical ultrasonic signal. Based on amplitude distribution, a typical signal can be modelled using heavy tails distribution. The efficient threshold level estimator [7] for this type of signals can be based on standard deviation $\sigma$ (STD)

$$
\hat{T}_{ij}^{\text{std}} = k\sigma = k\sqrt{\frac{1}{N_i - 1} \sum_{j=1}^{N_i} (T_{ij} - \bar{T})^2},
$$

(6)

and standard deviation with mean value (MEAN+STD)

$$
\hat{T}_{ij}^{\text{meanstd}} = \sqrt{\mu_i + k\sigma_i},
$$

(7)

where $N$ is the length of each set of detailed coefficients $j$ at level $i$, $k$ is the coefficient depending on signal crest factor and $\mu_i$ is the mean value.

Relations between parameters $k$, $\alpha$, and $\tau$ were studied by means of simulation. The decomposition level was experimentally set to four. To the simulated ultrasonic signal we added the fault echo amplitude $A_o$ within 1–100 % of the initial echo amplitude.

The performance of the denoising was evaluated by two parameters. The first parameter is based on signal-to-noise enhancement and can be expressed as

$$
SNRE = 10 \log \frac{P_1}{P_2},
$$

(8)

where $P_1$ is the power of the simulated noise and $P_2$ is the power of the noise after de-noising. Another parameter evaluates fault echo suppression in terms of amplitude decreasing and shape corruption. The parameter can be expressed as

$$
K_c = R_{A_o A_d}(0)(1 - \frac{A_o - A_d}{A_o}),
$$

(9)

where $R$ is the cross-correlation, $A_o$ and $A_d$ are the maximal fault echo amplitudes before and after de-noising. In this study, combinations have been computed with the different threshold levels, threshold rules and mother wavelets. Based on these simulations, the best results were obtained with the discrete
Meyer mother wavelet, hard thresholding and threshold level based on standard deviation. The following graphs (see Fig. 9 and Fig. 10) present the dependency of the parameters $SNRE$ and $K_c$ on the fault echo amplitude $A_a$ and coefficient $k$ for hard thresholding.

![Graph 9](image1)

**Fig. 9.** Evaluation of DWT de-noising with $K_c$, using the hard threshold rule a) STD, b) MEAN + STD.

![Graph 10](image2)

**Fig. 10.** Evaluation of DWT de-noising with $K_c$ using hard threshold rule a) STD, b) MEAN + STD.

The detailed assessment of all threshold rules and mother wavelets for both threshold levels STD and MEAN+STD is shown in Tab. 2 and Tab. 3.

As can be seen in Tab. 2, the minimal fault echo amplitude that can be efficiently detected is 5%. In this case the fault echo is almost without changes (parameter $K_c = 0.981$). Similar results were obtained with compromising (see Tab. 3) thresholding. Custom thresholding does not provide suitable results.

The detailed results of the wavelet transform de-noising methods are illustrated in Fig. 11. The graphs show that the best performance is from the wavelet packet de-noising method with the Daubechies mother wavelet of order 6. The SNRE is between 23 and 45 dB, depending on the fault echo amplitude. The DWT de-noising method also has high SNRE, and the shape of the SNRE curves is very similar to the WP method. The SNRE values for SWT are higher than the WP and DWT values, but in the case of SWT the
Fig. 11. SNRE for different fault echo amplitudes - top left: db4; top right: db6; bottom left: haar; bottom right: dmey.

Amplitudes of the back-wall and fault echoes are distorted even for high amplitudes. This is an undesirable phenomenon caused by the non-linear nature of wavelet based de-noising.

Amplitude preservation is an essential requirement for ultrasonic signal processing, because the amplitude of the fault echo characterizes the size of the flaw. Flaw detection is the main reason for using de-noising methods, and the different de-noising methods are evaluated to find which method is appropriate for finding the minimum fault echo amplitude. The Tab. 4 shows the minimum fault echo amplitudes necessary for successful de-noising and flaw echo detection.

It can be seen that the best results were obtained with the discrete Meyer mother wavelet, with successful detection of 5 % fault echo amplitude. On the other hand, the discrete Meyer mother wavelet has lower SNRE values. When the flaw echo detection is preferred, the discrete Meyer mother wavelet is better for de-noising with the WP method. The SNRE is from 15 to 40 dB.

The results of the thresholding rules with wavelet packet de-noising and the discrete Meyer mother wavelet with two levels, 10 % and 50 %, are evaluated in Tab. 5.

The SNRE of the common thresholding rules has a maximum value of 3.2 [dB].
Examples of different de-noising methods with the application of different mother wavelets with 9% of fault echo are shown in Fig. 12.

![Filtered ultrasonic signal with 9% fault echo](image)

Fig. 12. Filtered ultrasonic signal with 9% fault echo - top left: SWT, haar; top right: WP, db6; bottom left: WP, dmey; bottom right: DWT, dmey.

Tab. 4 and Fig. 12 show that with the amplitude fault echo 9% of back-wall echo, the db6, db4 and haar mother wavelets make it impossible to detect the fault echo. On the other hand, dmey mother wavelet de-noising works from 7% of back-wall echo.

As was mentioned above, the commonly used techniques for ultrasonic signal de-noising are split spectrum processing (SSP) and non-causal IIR and FIR filters. For our comparison, we performed all these methods (see Fig. 14). In the case of the SSP technique, the SSP minimization algorithm [2] was used. Both IIR and FIR filters were designed based on the known transducer frequency response. The highest SNRE is about 14 dB with the SSP method. This is much lower than the WP de-noising method. In order to make a comparison of the proposed methods, a real ultrasonic signal was also used. The signal was measured on a sample of a grainy material used for airplane engines. Two parts of the grainy material 10 mm in thickness were welded. Before welding, drilled circular artificial flaws, on average about 0.7 mm in diameter, were created in one part of the grainy material. To measure the artificial flaw we used a transducer with a center frequency of 25 MHz. The measured and filtered ultrasonic signal are demonstrated in Fig. 14. The signal was filtered with the wavelet packets de-noising method using the discrete Meyer mother wavelet, hard thresholding, and threshold level based on standard deviation.
The measured backscattering and electronic noise was efficiently suppressed without changes in fault and back-wall echo amplitude.

The wavelet transform is similar to correlation analysis; the result is expected to be maximal when the input signal fits the shape of mother wavelet. From the set of available wavelet functions the Meyer wavelet provides the best fit to an ultrasonic echo, consequently the denoising using this wavelet led to the highest noise reduction performance. Wavelet packets offer finer frequency decomposition over discrete wavelet transform (for $L$ levels of decomposition).
the wavelet packets transform produces $2L$ sets of coefficients as opposed to $(L + 1)$ sets for the discrete wavelet transform), thus the thresholding process can be more selective. Major impact on overall denoising performance has the threshold estimator. Ultrasonic signals constitute a narrow set of signals with \textit{a priori} known amplitude distribution. Custom estimators e.g. (6), based on \textit{a priori} information perform better than estimators developed for common signals.

4 Conclusions

This paper reports on a comparison of the discrete wavelet transform, the discrete stationary wavelet transform and the wavelet packets de-noising methods. These methods are compared on a simulated ultrasonic signal with different sizes of fault echo using signal-to-noise ratio enhancement and filtering efficiency. The best-performing method was wavelet packet de-noising, with SNRE within 15 to 40 [dB]. The most effective of the set of available mother wavelet functions was the discrete Meyer wavelet. With the proposed method, a flaw with the relative amplitude of fault echo 7% of the back-wall echo can be reliably detected.

Acknowledgements

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References


REFERENCES


5 Figure captions

(1) Illustration of the noise suppression procedure based on the discrete wavelet transform (DWT), only two decomposition levels are depicted, HP and LP are high-pass resp. low-pass filters, ↑ 2 and ↓ 2 stand for up-sampling resp. down-sampling.

(2) Backscattering noise - left, frequency spectrum - right.

(3) Typical ultrasonic noise - left, frequency spectrum - right.

(4) Simulated ultrasonic signal containing the back-wall echo and backscattering noise.

(5) Microscopic image of the grainy material.

(6) Simulated ultrasonic signal with a fault echo.

(7) Examples of mother wavelets with projected echo - a) Daubechie 4 (db4), b) Daubechie 6 (db6), c) Haar (haar), d) discrete Meyer (dmey).

(8) Principle of custom and compromising thresholding - a) custom thresholding $T = \text{const.}, \ 0 < \alpha < 1$, b) compromising thresholding $\alpha = \text{const.}, \ 0 < \tau < T$.

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(10) Evaluation of DWT de-noising with $K_c$ using hard threshold rule a) STD, b) MEAN + STD.

(11) SNRE for different fault echo amplitudes - top left: db4; top right: db6; bottom left: haar; bottom right: dmey.

(12) Filtered ultrasonic signal with 9% fault echo - top left: SWT, haar; top right: WP, db6; bottom left: WP, dmey; bottom right: DWT, dmey.

(13) SNRE evaluation for SSP, IIR and FIR methods.

(14) Real ultrasonic signal from coarse-grained material - left, filtered real ultrasonic signal - right.

(15) Real ultrasonic signal from coarse-grained material (B-scan) - left, filtered real ultrasonic signal - right.
Table 2
Performance of hard thresholding (top STD threshold level estimator, under double line MEAN+STD threshold level estimator)

<table>
<thead>
<tr>
<th>Mother wavelet</th>
<th>db2</th>
<th>db4</th>
<th>db6</th>
<th>dmey</th>
</tr>
</thead>
<tbody>
<tr>
<td>max. $K_c$ [-]</td>
<td>0.994</td>
<td><strong>0.989</strong></td>
<td>0.978</td>
<td>0.981</td>
</tr>
<tr>
<td>$SNRE$ [dB]</td>
<td>25.97</td>
<td><strong>37.76</strong></td>
<td>35.18</td>
<td><strong>37.59</strong></td>
</tr>
<tr>
<td>min. $A_a$ [%]</td>
<td>9</td>
<td>7</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>min. $k$ [-]</td>
<td>1.3</td>
<td>2.0</td>
<td>1.1</td>
<td>1.4</td>
</tr>
<tr>
<td>max. $K_c$ [-]</td>
<td>0.967</td>
<td>0.976</td>
<td>0.966</td>
<td>0.984</td>
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<tr>
<td>$SNRE$ [dB]</td>
<td>24.70</td>
<td>24.59</td>
<td>19.33</td>
<td>19.72</td>
</tr>
<tr>
<td>min. $A_a$ [%]</td>
<td>13</td>
<td>9</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>min. $k$ [-]</td>
<td>1.3</td>
<td>4.5</td>
<td>1.4</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Table 3
Performance of compromising thresholding (top STD threshold level estimator, under double line MEAN+STD threshold level estimator)

<table>
<thead>
<tr>
<th>Mother wavelet</th>
<th>db2</th>
<th>db4</th>
<th>db6</th>
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<tr>
<td>max. $K_c$ [-]</td>
<td>0.991</td>
<td><strong>0.991</strong></td>
<td>0.989</td>
<td><strong>0.991</strong></td>
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<tr>
<td>$SNRE$ [dB]</td>
<td>26.76</td>
<td><strong>32.88</strong></td>
<td>31.09</td>
<td><strong>31.83</strong></td>
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<tr>
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<td>6</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>min. $k$ [-]</td>
<td>1.3</td>
<td><strong>2.0</strong></td>
<td>1.1</td>
<td>1.4</td>
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<tr>
<td>min. $\alpha$ [-]</td>
<td>0.16</td>
<td>0.22</td>
<td>0.18</td>
<td>0.20</td>
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<tr>
<td>max. $K_c$ [-]</td>
<td>0.959</td>
<td>0.967</td>
<td>0.982</td>
<td>0.976</td>
</tr>
<tr>
<td>$SNRE$ [dB]</td>
<td>26.70</td>
<td>32.98</td>
<td>30.34</td>
<td>30.81</td>
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<tr>
<td>min. $A_a$ [%]</td>
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<td>10</td>
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<tr>
<td>min. $k$ [-]</td>
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<td>4.5</td>
<td>1.4</td>
<td>1.4</td>
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</table>
Table 4
Minimal fault echo amplitude for successful detection.

<table>
<thead>
<tr>
<th>Mother wavelet/Method</th>
<th>DWT [%]</th>
<th>WP [%]</th>
<th>SWT [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>haar</td>
<td>5</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>db4</td>
<td>7</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>db6</td>
<td>9</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>dmey</td>
<td>5</td>
<td>7</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 5
SNRE of different thresholding methods for noise reduction.

<table>
<thead>
<tr>
<th>Method</th>
<th>heursure</th>
<th>sqtwolog</th>
<th>rigsure</th>
<th>minimaxi</th>
<th>STD threshold level estimator</th>
</tr>
</thead>
<tbody>
<tr>
<td>WP - 10%</td>
<td>3.09 dB</td>
<td>3.08 dB</td>
<td>3.14 dB</td>
<td>3.14 dB</td>
<td>17.03 dB</td>
</tr>
</tbody>
</table>